5. (a) If $u = f(r,s)$, $r = x+y$, $s = x-y$, prove that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y} = 2\frac{\partial u}{\partial r}$. 2½

(b) Expand $x^4 + x^2y^2 - y^4$ about the point $(1, 1)$ upto the terms of the second degree. 2½

**Unit III**

6. Show that the function $f(x, y) = \sqrt{x^2+y^2}$ is continuous at $(0, 0)$ but $f_x$, $f_y$ do not exist at $(0, 0)$. 5

7. Find the maximum and minimum distance of the point $(3, 4, 12)$ from the sphere $x^2+y^2+z^2=1$ using Lagrange's method of multipliers. 5

**Unit IV**

8. (a) Evaluate $\int_0^\infty \frac{x^4\left(1+x^5\right)}{(1+x)^{15}}dx$. 2½





# Subject Code—52008

## B. A. EXAMINATION

(Reappear/Main) (Batch 2018 Onwards)

(Third Semester)

MATHEMATICS

BAMH-201

Paper–A

Advanced Calculus

*Time* : 3 *Hours* *Maximum Marks* : 25

**Note** : Attempt *Five* questions in all. Q. No. **1** is compulsory. All questions carry equal marks.

**(Compulsory Question)**

1. (a) If a function $f$ is uniformly continuous on $[a, b]$, then prove that it is continuous on $[a, b]$.

(b) Verify Rolle's theorem for the function $f(x)=\sqrt{4-x^2}$ in $[-2, 2]$.

(c) If $u=\log\left(x^3+y^3-x^2y-xy^2\right)$, show that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=2(x+y)^{-1}$.

(d) Show that the function :

$$f(x, y)=(y-x)^4+(x-2)^4,$$

has a minimum at $(2, 2)$.

(e) Evaluate the integral $\iint_R x\,dx\,dy$, where :

$R=[(x, y): 0 \le x \le a, 0 \le y \le x]$. **5×1=5**

**Unit I**

**2.** (a) Show that $f(x)=\sqrt{x}$ is uniformly continuous on $[1, 2]$. **2½**

(b) Verify Lagrange's mean value theorem for the function $f(x)=x+\frac{1}{x}$ in $[1, 3]$. **2½**



**3.** (a) By Maclaurin's expansion show that :

$$\cos x=1-\frac{x^2}{2!}+\frac{x^2}{4!}-....+(-1)^n\frac{x^{2n}}{2n!}+$$

$$(-1)^{n+1}\frac{x^{2n+1}}{(2n+1)!}\sin(\theta x).$$ **2½**

(b) Evaluate $\lim_{x\to 0}\left(\frac{\sinh x}{x}\right)^{\frac{1}{x^2}}$. **2½**

**Unit II**

**4.** (a) Prove that the function $f$ defined by :

$$f(x, y)=\begin{cases} y\sin\frac{1}{x}, & \text{if } x\neq 0 \\ 0, & \text{if } x=0 \end{cases}.$$

is continuous at the origin. **2½**

(b) If $u=\tan^{-1}\frac{x^2+y^2}{x+y}$, show that :

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{1}{2}\sin 2u.$$ **2½**

(b) Evaluate $\iint_A \sqrt{4x^2 - y^2}\, dxdy$, where A is the triangle bounded by the lines $y = 0$, $y = x$, $x = 1$. **2½**

**9.** (a) Find the area of the region in the first quadrant which is bounded by the parabola $y^2 = 4ax$ and the line $x = 2a$. **2½**

(b) Evaluate the integral :

$$\int_0^{\infty}\int_x^{\infty} \frac{e^{-y}}{y}\, dy\, dx$$

by changing the order of integration. **2½**