Section III

6. (a) Obtain the Fourier series expansion of the function $f(x) = |\cos x|$ in $(-\pi, \pi)$. **3**

(b) Find a series of cosines of multiples of x which will represent $f(x) = x \sin x$ in $(0, \pi)$. **2**

7. (a) Find the Fourier series expansion for the function $f(x)$ in $(0, 2\pi)$ defined as : **3**

$$f(x) = \begin{cases} x & 0 < x < \pi \\ 2\pi - x & \pi < x < 2\pi \end{cases}$$

Hence deduce that :

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots = \frac{\pi^2}{8}.$$

(b) Find the Fourier series expansion for $f(x)$ if : **2**

$$f(x) = \begin{cases} 0 & -2 < x < -1 \\ k & -1 < x < 1 \\ 0 & 1 < x < 2 \end{cases}$$

Roll No. .... **Exam Code : M-21**

Subject Code—52897

B. A. EXAMINATION

(Main) (Batch 2018)

(Fifth Semester)

MATHEMATICS

BAMH-302(i)

Sequence and Series

Time : 3 Hours *Maximum Marks :* 28

Note : Attempt *Five* questions in all. Q. No. **1** is compulsory.

1. (i) Show that the set N of natural number has no limit point.

(ii) Prove that $\overline{(\overline{A})} = A$.

(iii) Discuss the convergence of the series $\sum_{n=1}^{\infty} \frac{n}{n+1}$.

(iv) Show that the series $\sum_{n=1}^{\infty}(-1)^n.n$ oscillates infinitely.

(v) State Raabe's test.

(vi) State D'Alembert ratio test.

(vii) Define Primitive of a function.

(viii) Use fundamental theorem to compute $\int_0^{\pi/3}\cos x dx$. **8**

Section I

2. (a) Every non-empty subset of real numbers which is bounded below has a real number as its infimum. **3**

(b) A point p is a limit point of a set A if and if every neighbourhood of p contains infinitely many points of A. **2**

3. (a) Prove that every convergent sequence is bounded but not conversely. **3**

(b) Prove that a monotonically increasing sequence $\langle a_n \rangle$ which is bounded above converges to its least upper bound. **2**

Section II

4. (a) Test the convergence of the series : **3**

$$\frac{1}{1.2.3.}+\frac{x}{4.5.6}+\frac{x^2}{7.8.9}+.......$$

(b) Test the convergence of the series : **2**

$$x+\frac{2^2.x^2}{2!}+\frac{3^3.x^3}{3!}+\frac{4^4.x^4}{4!}+........$$

5. (a) If $\sum_{n=1}^{\infty}a_n$ is an absoutely convergent series, then the series of its positive terms and the series of its negative terms are both convergent. **3**

(b) Show that $\sum_{n=1}^{\infty}\frac{1}{n^p}.a_n\,(p\geq 0)$ is convergent if $\sum_{n=1}^{\infty}a_n$ is convergent. **2**

Section IV

8. (a) Prove that if $\sum_{r=1}^{n} f(\xi_r)\delta_r$ exists as $\|p\| \to 0$ and is equal to I, then f is Riemann integrable on $[a, b]$ and $I = \int_a^b f dx$. **3**

(b) State and prove fundamental theorem of integral calculus. **2**

9. (a) If a function f is defined on $[0, a]$ $a > 0$ by $f(x) = x^3$, then show that f is Riemann integrable on $[0, a]$ and evalaute $\int_0^a f dx$. **3**

(b) Prove that : **2**

$$\frac{\pi^2}{9} \le \int_{\pi/6}^{\pi/2} \frac{x}{\sin x} dx \le \frac{2\pi^2}{9}.$$

Section IV

8. (a) Prove that if $\sum_{r=1}^{n} f(\xi_r)\delta_r$ exists as $\|p\| \to 0$ and is equal to I, then f is Riemann integrable on $[a, b]$ and $I = \int_a^b f dx$. **3**

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$$\frac{\pi^2}{9} \le \int_{\pi/6}^{\pi/2} \frac{x}{\sin x} dx \le \frac{2\pi^2}{9}.$$