Section IV

8. (a) If $A + iB = \cos(x + iy)$, then prove that $(1+A)^2 + B^2 = (\cosh y + \cos x)^2$. **2.5**

(b) Prove that :

$$\left(i^i\right)^i = \cos(4n+1)\frac{\pi}{2} - i\sin(4n+1)\frac{\pi}{2}.$$ **2.5**

9. (a) Find the sum of series :

$\sin\alpha + \sin 2\alpha + \sin 3\alpha + ...$ to n terms and deduce the sum of the series $1 + 2 + 3 + ... n = \dfrac{n(n+1)}{2}$. **2.5**

(b) Find α if :

$$\sin^{-1}\frac{1}{\sqrt{5}} + \cos^{-1}\alpha = \frac{\pi}{4}.$$ **2.5**

Subject Code—52899

B. A. EXAMINATION

(Batch 2018) (Main)

(Fifth Semester)

MATHEMATICS

BAMH-303 (i)

Number Theory and Trigonometry

Time : 3 *Hours* *Maximum Marks :* 24

Note : Attempt *Five* questions in all. Q. No. **1** is compulsory which is of 4 marks. remaining all questions carry equal marks.

(Compulsory Question)

1. (a) Find a such that $a \equiv 8 \pmod 3$. **0.5**

(b) Evaluate $\phi(10000)$. **0.5**

(c) State Fermat' theorem. **0.5**

(d) State Wilson's theorem. **0.5**

(e) Find the principal and general value of Log (–5). **0.5**

(f) Resolve $\log(1-\cos 2\theta + i\sin 2\theta)$. **0.5**

(g) Show that 16! + 1 is divisible by 17 for any integer n. **0.5**

(h) Prove that $\tan(ix) = i\tanh x$. **0.5**

Section I

2. (a) Find all solutions in integers of the linear Diophantine equation $40x + 63y = 521$. **2.5**

(b) Solve the congruence $6x \equiv 3 \pmod{75}$. **2.5**

3. (a) Find the remainder 3^{181} is divided by 17. **2.5**

(b) Find the least positive integer x such that :

$x \equiv 1 \pmod 3 \; x \equiv 2 \pmod 4 \; x \equiv 3 \pmod 5$. **2.5**

Section II

4. (a) Find the number of positive integers ≤ 10000 that are co-prime to 10000. **2.5**

(b) Write a complete residue system modulo 11 composed entirely of multiples of 2. **2.5**

5. (a) Find the number and sum of positive divisors of 3000. **2.5**

(b) Define Mobius function and calculate its value for 5000. **2.5**

Section III

6. (a) Determine whether $x^2 \equiv 2 \pmod{61}$ is solvable or not. **2.5**

(b) Find the quadratic residue and quadratic non-residue of 13. **2.5**

7. (a) State and prove Quadratic Reciprocity law. **2.5**

(b) Evaluate $\left(\frac{7}{13}\right)$ using Gauss Lemma. **2.5**